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How the subsidy and its components are derived from cash flow observations

Direct loans and loan guarantees have a common set of subsidy components:

- Interest
- Fee
- Default, net of recoveries
- All other

For each component, the subsidy percentage is calculated by dividing the present value of certain cash flow observations (described below) by loan disbursements. The quotient is multiplied by 100 and rounded to two decimal places.

The total subsidy percentage is the sum of the four components.

A few definitions

"Cash flow observations" refers to a stream of cash transactions between a Federal credit program and the public. For direct loan programs, cash flow observations might refer to loan disbursements, contractual principal or interest payments by borrowers, adjustments for defaults or recoveries, prepayments, or fees received. In each case, the cash flow observations are one or more cash inflows or outflows of a particular type.

"*Present value*" or "*net present value*" is the value today of a dollar in some future period, adjusted for the time value of money. When used in regard to a series of payments, "present value" is the sum of the present value of each payment. It is derived by multiplying the payment in each period by a present value factor for that period. "*Net present value*" is sometimes used in regard to a series that includes both payments and receipts.

The "*present value factors*" are factors, based on assumptions about interest rates, that, when multiplied by the dollar amount for that period, yield the value in today's dollars. See "*Description of the "basket-of-zeros" discounting method and the derivation of present value factors from the yield curve"* for details on the derivation of present value factors. Interested readers might also consult "*How the CSC selects a basis for computing present value factors*."

The "*basket-of-zeros*" discounting method defines the present value of a series of payments as the value today of a collection of zero-coupon bonds that, at maturity, exactly match the cash flow observations. The derivation of present value factors from the yield curve is described in a related paper, *"Description of the "basket-of-zeros" discounting method and the derivation of present value factors from the yield curve."*

Derivation of the subsidy components from cash flow observations

The subsidy components, in dollar terms, are calculated as described below. To convert them into percentage terms, the dollar amounts are divided by loan disbursements. Then, the result is multiplied by 100 and rounded to two decimal places.

The total subsidy cost is the sum of the four components.

> Financing/interest subsidy costs are defined as the portion of the subsidy attributable to subsidizing the borrower's interest costs by charging lower rates than the discount rate (in certain direct loan programs) or by direct interest subsidy payments (in certain loan guarantee programs). For direct loans, this is calculated as the excess of the amount of the loans disbursed over the present value of the interest and principal payments required by the loan contracts. For loan guarantees, this is calculated as the present value of estimated interest supplement payments, before adjustment for defaults.
>
> Defaults, net of recoveries, subsidy cost, defined as the portion of the subsidy attributable to defaults, net of recoveries. It is calculated as the sum of discounted cash flow observations for defaults and recoveries.
>
> Fee subsidy cost, defined as the portion of the subsidy percentage attributable to up-front and annual fees paid to the government. Because these fees are inflows to the Government, this subsidy component makes the total subsidy either less positive or more negative. It is calculated as the sum of the discounted fee-related cash flow observations, before adjustment for defaults.
>
> Other subsidy costs, defined as the residual subsidy cost not attributed to financing, defaults net of recoveries, or fees. It is calculated as a residual.

Technical considerations

The subsidy percentage and its components are derived from several streams of cash flow observations, which are discounted to the "time of disbursement" and aggregated. A technical description of how this is done is contained in the appendices to this paper:

Appendix A: Definitions from the Federal Credit Reform Act and Related Materials provides some important definitions and requirements.

Appendix B: Discounting to the "Time of Disbursement" describes the implementation of this requirement.

Appendix C: Specifications and examples of the pro-rata method describes this method for allocating aggregated data to disbursement periods.

Appendix D: Specifications and examples of the reverse-spendout method describes this method for allocating aggregated data to disbursement periods.

Changes from previous methods

The methods in the revised Credit Subsidy Calculator (CSC) are a significant departure from those used previously, though the effect of these changes on the subsidy estimates will generally be minor. The important differences are:

> The basket-of-zeros discounting method is used in the place of a constant discount rate for cash flows estimated to occur at different times.
>
> A single effective rate is computed to preserve the equality between the rates at which financing accounts earn or pay interest and the rate used for discounting.
>
> Cash flow observations may be prepared in monthly, quarterly, and semi-annual frequencies. Previously, all cash flow observations were in annual frequency only.
>
> Cash flows are *directly* discounted to the exact time of disbursement. Previously they were *indirectly* discounted to the time of disbursement by discounting cash flows and disbursements to the beginning of the fiscal year in which they occurred. The previous method was accurate when used with reverse-spendout discounting (which was used in the previous model) but is not accurate when the basket-of-zeros discounting method is used.

Appendix A:
Definitions from the Federal Credit Reform Act and Related Materials

The Federal Credit Reform Act

The Federal Credit Reform Act of 1990 (FCRA) made fundamental changes in the budgetary treatment of direct loans and loan guarantees. FCRA shifted the budget basis from the amount of cash expected to flow into or out of the Treasury to the estimated subsidy costs of the loans or guarantees.

As defined by the act, the subsidy cost of a direct loan or a loan guarantee is the estimated long-term cost to the Federal Government calculated on a present value basis, excluding administrative costs. The subsidy cost of a direct loan or a loan guarantee is calculated by projecting the related cash flows to and from the government over the life of the loans and then discounting those cash flows back to the time of disbursement.

In 1997, section 502(5)(E) of the FCRA was amended to require the use of the basket-of-zeros discounting method. This method defines the present value of a series of payments as the price of a collection of zero-coupon bonds that, at maturity, exactly match the stream of payments in amount and timing. (See the article entitled "*Description of the "basket-of-zeros" discounting method and the derivation of present value factors from the yield curve*" for details on this method.) Compared to the former method, this method is more precise and will yield identical cost estimates for credit programs that have identical cash flows, even though the underlying loans may be of different maturity.

Cohorts and risk categories as the unit for subsidy calculation

OMB Circular A-11 defines a cohort as:

> …all direct loans or loan guarantees of a program for which a subsidy appropriation is provided for a given fiscal year…. For direct loans and loan guarantees for which a subsidy appropriation is provided for one fiscal year, the cohort will be defined by that fiscal year. For direct loans and loan guarantees for which multi-year or no-year appropriations are provided, the cohort is defined by the year of obligation. Direct loans and loan guarantees that are made from supplemental appropriations will be recorded in the same cohort as those that are funded in annual appropriation acts. These rules apply even if the direct loans or guaranteed loans are disbursed in subsequent years. (OMB Circular A-11, *Preparation and Submission of Budget Estimates*, Transmittal Memorandum No. 72, July 12, 1999, page 283.)

OMB Circular A-11 defines risk categories as:

… subdivisions of a cohort of direct loans or loan guarantees into groups that are relatively homogeneous in cost, given the facts known at the time of obligation or commitment. They are developed by agencies in consultation with the OMB representative with primary budget responsibility for the credit account. The number will depend on the size of the difference in subsidy cost between categories and the ability to predict it statistically based on facts known at origination.

Risk categories will group all direct loans or loan guarantees within a cohort that share characteristics predictive of defaults and other costs. They may be defined by characteristics or combinations of characteristics of the loan, the project financed, and/or the borrower. Examples of characteristics or indicators that may predict cost include:

> The loan-to-value ratio;
>
> The relationship between the loan interest rate and relevant market rates;
>
> Type of school attended for education loans;
>
> Country risk categories for international loans; and
>
> Various asset or income ratios.

Statistical evidence must be presented, based on historical analysis of program data or comparable credit data, concerning the likely costs of defaults, other deviations from contract, or other costs that are expected to be associated with the loans in that category. (OMB Circular A-11, *Preparation and Submission of Budget Estimates*, Transmittal Memorandum No. 72, July 12, 1999, pages 289-90.)

Appendix B:
Discounting to the "Time of Disbursement"

The Federal Credit Reform Act (Section 502(5)(B) and (C)) requires discounting all cash flow observations to the point of disbursement of the loan to the borrower:

> (B) The cost of a direct loan shall be the net present value, **at the time when the direct loan is disbursed**, of the following estimated cash flows:
>
> > (i) loan disbursements;
> > (ii) repayments of principal; and
> > (iii) payments of interest and other payments by or to the Government over the life of the loan after adjusting for estimated defaults, prepayments, fees, penalties, and other recoveries;
>
> including the effects of changes in loan terms resulting from the exercise by the borrower of an option included in the loan contract.
>
> (C) The cost of a loan guarantee shall be the net present value **at the time when the guaranteed loan is disbursed**, of the following estimated cash flows:
>
> > (i) payments by the Government to cover defaults and delinquencies, interest subsidies, or other payments;
> > (ii) payments to the Government including origination and other fees, penalties, and recoveries;
>
> including the effects of changes in loan terms resulting from the exercise by the guaranteed lender of an option included in the loan guarantee contract, or by the borrower of an option included in the guaranteed loan contract. (*Federal Credit Reform Act of 1990*, Section 502(5), "Definitions" Boldface added for emphasis.)

In combination with the requirement to use the "basket-of-zeros" method for discounting, the results of discounting to the "time of disbursement" as opposed to, say, the beginning of the first fiscal year of the cohort, can be substantial. Consider the following example:

> A series of three loans are made at the beginning of three successive years. Each loan has a balloon payment after three years with 5 percent interest, compounded annually. The cash flows and present value factors (which are calculated from the interest rates in the economic assumptions for the FY 1999 Budget) are:

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Loan 1:						
Disbursement…	100.00					
Repayment…...				115.76		
Loan 2:						
Disbursement…		100.00				
Repayment…...					115.76	
Loan 3:						
Disbursement…			100.00			
Repayment…...						115.76
Totals:						
Disbursement…	100.00	100.00	100.00			
Repayment…...				115.76	115.76	115.76
PV factors	1.00000	.950495	.900567	.852296	.805735	.761002

When discounted to the point of disbursement, the present values of the cash flows and subsidies for the loans are equal, as follows:

PV disbursement = 100.00 • 1.000000 = 100.00
PV repayment = 115.76 • 0.852296 = 98.66
Subsidy = 100.00 - 98.66 = 1.34 (also 1.34 percent)

The subsidy for the three loans combined would be:

3 • 1.34 = 4.02 (in dollars)

and

4.02 / 300.0 = **1.34 percent (let's call this result "A")**

If, instead, the aggregated cash flows were discounted to the beginning of year 1, the result would be:

PV disbursements = 100.00 • 1.000000 + 100.00 • 0.950495
+ 100.00 • .900567
or 285.11

PV repayments = 115.76 • 0.852296 + 115.76 • 0.805735 + 115.76 • 0.761002

or 280.02

The subsidy would be:

285.11 - 280.02 = 5.09 (in dollars)

and

5.09 / 285.11 = **1.78 percent (let's call this result "B")**

Why do the subsidies in result “A” and “B” differ? In “A,” all repayments are discounted to the time of disbursement using a present value factor derived from the 3-year spot rate. In “B,” each repayment is discounted to the time of disbursement using a present value factor derived from the 3-year forward rate.

Which calculation is correct? If all discounting were done on the basis of the yield curve prevailing at the time when the cohort began, result “B” would be correct. Under this assumption, forward rates are the implicit forecast for future rates and, with upward-sloping yield curves, are assumed, generally, to be rising over time. (Forward rates, implicitly, assume that the upward slope in the yield curve arises entirely from expectations that rates will rise over time. In fact, yield curves may have an upward slope for several reasons other than such expectations, including the increased risk inherent in longer-term lending, market segmentation, time preferences, and so forth.)

But Federal credit programs do not use the forward rates as a forecast of future rates. Instead, future interest rates are assumed explicitly for budget purposes. In particular, the yield curve assumed to prevail at the beginning of the cohort is assumed to prevail in all subsequent periods for the preparation of budget estimates. Later, when actual rates are available, they are substituted for the assumed rates.

Given this explicit assumption about future rates, the calculation that gave result “A” is appropriate for estimates of credit subsidies in prospective lending and is used by the CSC.

Considerations in implementing the “time of disbursement” provision

A useful implementation of the “time of disbursement” provision must deal with two items:

> **Timing**. The amount by which a payment should be discounted depends on the span of time between the time when the loan was disbursed and the time when the payment was made. If a payment occurs exactly one year after disbursement, it should be discounted by using a present value factor for one year, *exactly*. To make this calculation, cash flow estimates must be associated with a particular disbursement and the time of that disbursement must be known.
>
> **Multiple disbursements**. Few, if any, Federal credit programs make individual loans in isolation. For virtually all programs, the authority to make loans is sufficient to make many loans and often over a period of more than one year. Thus, the relationship between the stream of loan disbursements and the stream of inflows and outflows related to those loans is a “many-to-many” relationship. In these circumstances a method is needed to associate specific inflows and outflows with disbursements made at a specific time. Without a method for doing so, it would be impossible to determine the relationship between the time of disbursement and the time a payment is made.

The methods used in the CSC for these considerations are described in the following sections.

Timing considerations

An individual set of cash flow estimates may be provided in any of the following frequencies:

Annual
Semi-annual
Quarterly
Monthly.

Within any of these frequencies, cash flow estimates may be specified as taking place at the:

Beginning of the period
Middle of the period
End of the period.

In addition, each set of cash flow estimates has a specific starting date, which, by default, is the date when the authority is first available to make or guarantee loans. These start dates are translated into an "elapsed time" value in twice-monthly units. For example, October 1 of the year when authority first becomes available would have an "elapsed time" of zero. Twelve months later, on September 30 of the same fiscal year, the "elapse time" would be 24.

Finally, an "offset" is computed based on the relationship between the "elapsed time" of the beginning of the cash flow estimates and the "elapsed time" of the disbursement to which these cash flow estimates are attributed (more on how this is determined below). For example, if the disbursement took place in the middle of the first year ("elapse time" value of 12) and the first cash flow observation took place at year-end ("elapse time" of 24), the "offset" for that stream of cash flow observations would be 12.

Later, when the present value of the cash flows needs to be calculated, the offset and frequency are used to select present value factors to use in determining the present value. The present value factors are available on a twice-monthly frequency. Thus the offset is the index, on a twice-monthly basis, of the first present value factor to use (an index value of zero pertains to October 1 of the first fiscal year of the risk category).

Discounting, to the time of disbursement, then is calculated as follows:

$$\text{Present value} = \sum_{i=0}^{n-1} (X_i \bullet P_{(i \bullet f)+offset})$$

where:

X_i	Cash flow observation in period i, where the first period is 0, the second period is 1, and so forth, regardless of the frequency of the observations.
P_i	Present value factor for period i, where i is stated in twice-monthly intervals.
offset	is the “offset” as described above
f	Frequency, expressed in twice-monthly periods (e.g., annual frequency would be 24, monthly frequency would be 2)
n	Number of observations

Special handling is needed when the index of a present value factor is negative. When this happens, the index is negated and the reciprocal of the present value factor (for the negated index) is used instead.

This could occur, for example, if the disbursement by the private lender took place on the October 1 when the cohort began and the fees were collected on, say, the previous June 1. In this case, the present value of those fees would be higher than their cash value to reflect the interest that would accrue on them from June 1 to October 1.

Multiple disbursements

When all disbursements take place at the same time, there is no ambiguity in computing the “offset” as described above and in calculating a present value. When disbursements are made in two or more periods, an ambiguity arises regarding the point in time to which a cash flow observation should be discounted. For example, if a direct loan program makes disbursements at the beginning of two successive years, should the borrower’s payments received in the second year be discounted to the beginning of the first year or to the beginning of the second?

The CSC provides two methods for resolving the ambiguity:

Explicit association. Cash flow observations can be prepared in a way that specifically associates cash flow observations with the periods in which the underlying disbursement was made. This method is the most flexible and accurate approach for attributing cash flow observations to specific disbursement periods. It also requires more work on the part of analysts in preparing cash flow estimates.

Estimated association. When the agency does not associate cash flow observations with disbursements in a particular period, the CSC divides aggregate cash flow observations into separate groups that can be attributed to disbursements in individual periods. Two methods are available for doing this: the "pro-rata" method (described in Appendix C) and the "reverse-spendout" method (described in Appendix D). The "reverse spendout" method is more accurate, potentially, but does not work well in all instances. The "pro-rata" method is generally less accurate, but produces generally useful results in all circumstances. These methods are used as follows:

> If the spreadsheet contains a specification to "force" the pro-rata method, then the "pro-rata" method is used in all instances where aggregate data must be distributed by disbursement year.
>
> If the "force pro-rata" specification is not used, the CSC tries the "reverse-spendout" method first. If it succeeds (the test for success is given at the end of Appendix D), it is used. If not, the "pro-rata" method is used.

The use of alternative methods for associating cash flows with disbursement years raises some concerns about the consistency of results when small changes in cash flow observations cause the "reverse-spendout" method to fail and the "pro-rata" method to be used in its place. Such a switch in methods can result in a change in the results that are out of proportion to the change in input data. When comparability of methods is important, the specification can be used to force the use of the "pro-rata" method in all circumstances, even when the "reverse-spendout" method would be more accurate.

Appendix C
Specifications and examples of the pro-rata method

This appendix describes the pro-rata method that is used to associate cash flow observations with specific disbursement periods. This method is less refined than the "reverse-spendout" method, described in Appendix D. However, where the "reverse-spendout" method has limited applicability, this method will produce generally useful, if somewhat rough, approximations in all circumstances.

An example

Consider a loan guarantee program, with disbursements by private lenders, fees paid to the government, and payments from the government to the private lender when the borrower defaults, as follows:

	Year 1	Year 2	Year 3	Year 4	Year 5
Disbursements	15,000	15,000			
Upfront fees	150	150			
Annual fees	15	30	30	30	15
Default pmts				250	250

If our knowledge were limited to the values in the table, plus the fact that transactions take place at the beginning of each year, how would we attribute a portion of the annual fees, upfront fees, and default payments to each disbursement year? The basis for this allocation is the following:

Year 1: The fees are attributed to year 1 disbursements. They could not be attributed to disbursements not yet made.

Year 2: The upfront fees in year 2 are most likely to be related to year 2 disbursements. The annual fees are attributable to both years equally.

Year 3: The annual fees are attributable to both years equally.

Year 4: The annual fees are attributable to both years equally. The default payment is attributed to disbursements in year 1.

Year 5: For symmetry with the treatment of the first year, the annual fee is attributed to the year 2 disbursements only. The default payment is attributed to the last disbursement.

Cash flows, by disbursement year, would be estimated as follows:

	Year 1	Year 2	Year 3	Year 4	Year 5
Year 1:					
Disbursements	15,000				
Upfront fees	150				
Annual fees	15	15	15	15	
Default pmts				250	
Year 2:					
Disbursements		15,000			
Upfront fees		150			
Annual fees		15	15	15	15
Default pmts					250

The cash flows associated with year 1 disbursements would be discounted to the beginning of year 1; those associated with year 2 disbursements, to the beginning of year 2. Based on the specification that transactions occur at the beginning of the year and a constant discount rate of 6 percent, the present values would be determined as follows:

Year 1, upfront fees: 150 (they occur at the beginning of the year)

Annual fees: $(15 / 1.06^0) + (15 / 1.06^1) + (15 / 1.06^2) + (15 / 1.06^3)$

Or: 15.00 + 14.15 + 13.35 + 12.59

Or: 55.09

The present value of the year 2 cash flows would be identical. The upfront fee occurs at the time of disbursement and is therefore not discounted. The four receipts of annual fees occur at the time of disbursement, after one year, after two years, and after three years.

In this example, the pro-rata approach works reasonably well. It would not work well for all circumstances, however. See "Limitations of the pro-rata method" at the end of this appendix.

Description of the method

The pro-rata method attributes cash flow observations to disbursement years in proportion to the amounts disbursed in each year. The following example describes how pro-rata factors are developed to allocate cash flow observations from seven periods to disbursements in three periods.

The first step is to build a matrix from which distribution factors can be calculated. The matrix, with seven columns for the cash flow periods and three rows for the disbursement periods, is populated with the amounts disbursed. Clearly, cash flow observations in the first period could only be related to disbursements in the first period; cash flow observations in the second period could be attributed to disbursements in the first and second periods; and so forth. To prevent the disbursements in the first period from having excessive importance, a symmetrical pattern is used for the last cash flow observation: the last cash flow observation is attributed entirely to the last disbursement; the second to the last is attributed to the last two disbursements; and so forth. Thus, each disbursement will have a share of, at most, $N_{CF} - N_D + 1$ periods (N_{CF} is the number of cash flow observations and N_D is the number of disbursement periods). In our example, each disbursement would have a share of the five cash flow observations. The matrix would look like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6	CF 7
D 1	100.0	100.0	100.0	100.0	100.0		
D 2		100.0	100.0	100.0	100.0	100.0	
D 3			100.0	100.0	100.0	100.0	100.0
Total	100.0	200.0	300.0	300.0	300.0	200.0	100.0

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

(It may be useful to highlight the fact that disbursement amounts, rather than the amounts of cash flow observations, are used to compute distribution factors. A simple example will make the reason for this obvious. What would happen if a direct loan program made disbursements in three successive years in the amounts of 100, 0, 100?[1])

The above matrix is converted to distribution factors by dividing each cell by the column totals. The result looks like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6	CF 7
D 1	1.000	0.500	0.333	0.333	0.333		
D 2		0.500	0.333	0.333	0.333	0.500	
D 3			0.333	0.333	0.333	0.500	1.000

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

The cash flows attributable to disbursements in the first year would be estimated by multiplying the aggregate cash flows for all disbursements by the factors in the first row. Cash flow observations attributable to disbursements in the second and third years would be similarly obtained.

The results might look like this:

Original values	50.0	100.0	150.0	150.0	150.0	100.0	50.0

[1] Answer: None of the cash flows should be attributed to a year in which no disbursement occurred; however, if cash flow observations, rather than disbursements were used as allocation factors, that might happen.

Original values allocated to disbursement years:

D 1	50.0	50.0	50.0	50.0	50.0		
D 2		50.0	50.0	50.0	50.0	50.0	
D 3			50.0	50.0	50.0	50.0	50.0

(*Note*: Dn are the disbursement periods)

A couple of observations:

The cash flow observations for each year of disbursement are identical, except for the difference in the timing of disbursements;

The period of time from the first to the last cash flow observations is the same for each disbursement year, though all occur over a period of time shorter than the original values

An example with varying amounts disbursed

What happens when the disbursements are not in equal amounts? Consider an example where the disbursements in the three years are 100.0, 200.0, and 300.0. The matrix of disbursements-based weights would look like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6	CF 7
D 1	100.0	100.0	100.0	100.0	100.0		
D 2		200.0	200.0	200.0	200.0	200.0	
D 3			300.0	300.0	300.0	300.0	300.0
Total	100.0	300.0	600.0	600.0	600.0	500.0	300.0

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

And the matrix of factors (after dividing by the totals) would look like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6	CF 7
D 1	1.000	0.333	0.167	0.167	0.167		
D 2		0.667	0.333	0.333	0.333	0.400	
D 3			0.500	0.500	0.500	0.600	1.000

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

Results when some years are missing

This procedure can result in some patterns that might look odd at first. For example, consider five periods of cash flow observations and three periods of disbursements, in which disbursements are 100.0, 0.0, and 100.0.

The matrix of disbursements-based weights would look like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6
D 1	100.0	100.0	100.0	100.0		
D 2		0.0	0.0	0.0	0.0	
D 3			100.0	100.0	100.0	100.0
Totals	100.0	100.0	200.0	200.0	100.0	100.0

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

And the matrix of factors (after dividing by the totals) would look like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6
D 1	1.000	1.000	0.500	0.500		
D 2		0.0	0.0	0.0	0.0	
D 3			0.500	0.500	1.000	1.000

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

A diagonal matrix

Another example of irregular inputs occurs when there are as many periods of disbursements as periods of cash flow observations:

The matrix of disbursements-based weights would look like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6
D 1	100.0					
D 2		100.0				
D 3			100.0			
D 4				100.0		
D 5					100.0	
D 6						100.0
Totals	100.0	100.0	100.0	100.0	100.0	100.0

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

And the matrix of factors (after dividing by the totals) would look like this:

	CF 1	CF 2	CF 3	CF 4	CF 5	CF 6
D 1	1.000					
D 2		1.000				
D 3			1.000			
D 4				1.000		
D 5					1.000	
D 6						1.000

(*Note*: Dn are the disbursement periods; CFn are the periods with cash flow observations)

Treatment of truncated cash flow observations

If a set of cash flow observations has annual observations for loan disbursements and upfront fees and there are disbursements in three successive years, but upfront fees in just two years, there is a problem. What portion of the fees should be attributed to disbursements in the third year? When this occurs, the CSC attributes nothing to the third year.

In the general case, a diagonal matrix, equal in size to the number of cash flow observations and with values of 1.00 in the diagonal cells, will be used whenever the number of periods of cash flow observations is equal to or less than the number of periods of disbursements.

Limitations of the pro-rata method

The pro-rata method will produce generally reasonable results when cash flow observations have a relatively constant relationship to disbursements. The pro-rata method has increasingly severe limitations when the relationship between cash flow observations and disbursements varies over time.

In addition, interest payments could pose a special problem. The magnitude of interest payments is related to the loan balance outstanding, the interest rate charged, and the time since the last payment. With these considerations alone, we might choose a way to allocate interest payments that would differ from other kinds of payments. An example might help illustrate why this is so.

Consider a loan program in which disbursements are made in the middle of the first two periods. When the loan is disbursed, interest to the end of the first period is collected. Afterwards, interest payments are due at the end of each period. A balloon payment of principal is due with the final interest payment. Assuming an interest rate of five percent, the cash flow observations would look like this.

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Loan 1						
Disbursement…...	100.0					
Interest pmt……..	2.5	5.0	5.0	5.0	5.0	
Principal pmt…...					100.0	
Loan 2						
Disbursement…...		100.0				
Interest pmt……..		2.5	5.0	5.0	5.0	5.0
Principal pmt…...						100.0
Total						
Disbursement…...	100.0	100.0				
Interest pmt……..	2.5	7.5	10.0	10.0	10.0	5.0
Principal pmt…...					100.0	100.0

If the allocation approach described above were used, the allocation matrix would be:

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Disb. period 1	1.000	0.500	0.500	0.500	0.500	
Disb. period 2		0.500	0.500	0.500	0.500	1.000

and the interest payments would be distributed as follows:

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Disb. period 1	2.50	3.75	5.00	5.00	5.00	
Disb. period 2		3.75	5.00	5.00	5.00	5.00

The distribution in the second period is simply wrong.

The problem is that the allocation of interest payments (assuming identical interest rates) should be related to the outstanding balance of the loan and the time since the last interest payment. On that basis, the first loan would get a larger, rather than equal, share of the second year total.

The problem in making such an accommodation is the variety of loan contracts that might be considered. Loans may have prepayments of interest, grace periods, capitalization of unpaid interest, and sliding scales of interest rates charged. Each of these would call for a different kind of adjustment and some would require extensive specification of the terms of the loans.

A similar problem exists with principal payments. If the loan repayments are in constant amounts, with interest and principal, the principal payment of each successive payment is slightly larger. The allocation methods described above do not take that varying proportion into account, nor is there any plausible way to do so in the absence of the full details of the terms of the loan.

In both this case and the case of interest payments, agencies need to prepare disbursement period details, based on the characteristics of the program, rather than relying on the approximations used by the CSC when disbursement period details are not provided.

Appendix D
Specifications and examples of the reverse-spendout method

This appendix describes the reverse-spendout method that is used to associate cash flow observations with specific disbursement periods. This method is more refined than the "pro-rata" method, described in Appendix C, but is somewhat fragile, as explained below.

An example

Consider a loan guarantee program, with disbursements by private lenders. Fees are paid to the government on the basis of outstanding loan balances and decline over time. The fees paid to the government might look like this:

	Year 1	Year 2	Year 3	Year 4	Year 5
Disbursements	10,000	45,000			
Disb 1 balance	10,000	7,500	5,000	2,500	
Disb 1 fees	100	75	50	25	
Disb 2 balance		45,000	33,750	22,500	11,250
Disb 2 fees		450	338	225	112
Total fees	100	525	388	250	112

If these total fees were distributed using the pro-rata method (see Appendix C), the result would be:

	Year 1	Year 2	Year 3	Year 4	Year 5
Disb 1 fees	100	95	61	45	
Disb 2 fees		430	327	205	112

As explained in Appendix C, the pro-rata method is approximate and, in this case, the deviations from the actuals are obvious. Is there any better method to associating cash flow observations with disbursement years? Yes, though not necessarily in all circumstances.

The reverse-spendout method assumes some proportionality in the attribution of cash flow observations to disbursement years. Specifically, the proportionate relationship of the first period disbursement to the first period cash flow observation (which can be observed) is the same as the second period disbursement to its share of the second period cash flow observation (which cannot be observed). This "constant" relationship can be used to derive a more proportionate distribution of cash flow observations to disbursement years.

For the fees shown above, the "reverse-spendout" method would be applied as follows:

The first fee payment would be attributed to first period disbursements (same as the pro-rata method).

The portion of the second period fees attributed to the second period disbursements would be based on the ratio of the first period fees to first period disbursements:

$$[(100 / 10{,}000) \bullet 45{,}000] \quad \text{or} \quad 450$$

and the remainder, 75, would be attributed to the first period.

The portion of third year fees attributed to the second year of disbursements would be calculated in the same way:

$$[(75 / 10{,}000) \bullet 45{,}000] \quad \text{or} \quad 338$$

and the remainder, 50, would be attributed to the first period.

The process would be reversed for the final observations. The last fee observation would be attributed entirely to the last disbursement and a process similar to the above for the second to last.

The result would be as follows:

	Year 1	Year 2	Year 3	Year 4	Year 5
Disb 1 fees	100	75	50	25	
Disb 2 fees		450	338	225	112

This result exactly matches fees that were based on the declining loan balance.

Some problems

As mentioned earlier, the reverse-spendout method can be fragile. This is best shown by example. Let's take the example above and change a single number, in the second year, to one (obviously, this could be a data entry error; however, the CSC does not have enough information to evaluate data quality):

	Year 1	Year 2	Year 3	Year 4	Year 5
Disbursements	10,000	45,000			
Total fees	100	1	388	250	112

When the reverse-spendout method is applied, the portion of second year fees attributed to second year disbursements would be the same as above

[(100 / 10,000) • 45,000] or 450

and the residual, -449, would be attributed to the first year.

The portion of the third year fees attributed to the second year disbursements would be

[(-449 / 10,000) • 45,000] or -2,020.50

and the residual, 2,408.50, would be attributed to the first year.

When the allocation is finished, it would look like this:

	Year 1	Year 2	Year 3	Year 4	Year 5
Disb 1 fees	100	-449	2408.50	25	
Disb 2 fees		450	-2020.50	225	112

The reverse-spendout method fails here because the aggregate fees are *not* in proportion to disbursements. When a calculation is made assuming that they are, it should not be surprising that the results are unsatisfactory.

Test for applicability

The CSC assumes that the reverse-spendout method is applicable any time it results in distributions of cash flow observations to disbursement years in which all amounts distributed have the same sign as the aggregate amount. If not, the CSC uses the more robust (though less precise) pro-rata method.